Selection of Magnet Lamination Material and Thickness on the Basis of Eddy Currents

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by

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The procedure used in this study is a solution of the magnetic field diffusion equation for an idealized lamination having constant permeability μ, thickness d, and subject to the same surface field H_s on both sides of the lamination. It is assumed that the field averaged across the thickness has a triangular waveform of frequency f and amplitude (peak to trough) A. The diffusion equation is:

$$\frac{\partial^2 H}{\partial x^2} = \frac{\mu}{\rho} \frac{\partial H}{\partial t} \tag{1}$$

The fourier expansion of the triangular waveform is:

$$\bar{H} = \frac{A}{2} - \sum \frac{4A}{n^2 \pi^2} \cos n\omega t, \text{ n odd} \tag{2}$$

where $\omega = 2\pi f$. The solution to eq. (1) is

$$H = C_n \cosh (K_1 x) e^{in\omega t} + D_n \cosh (K_2 x) e^{-in\omega t} \tag{3}$$

where sinh terms are omitted since they do not satisfy the symmetric boundary condition. Since only cosh $n\omega t$ terms are in the expansion,

$$K_1 = (1 + i)/\delta n \text{ and } K_2 = (-1 + i)/\delta n \tag{4}$$

where $\delta_n = \sqrt{2\rho/n\omega\mu}$

The space average of H is $(1/d)\int_{-d/2}^{d/2} H\,dx$. If this is matched to the fourier expansion of the time, it is found that

$$C_n = \frac{Ad(1+i)}{n^2\pi^2\delta_n \sinh\,[(1+i)d/2\delta n]} \tag{5}$$

$$D_n = \frac{Ad\,(-1+i)}{n^2\pi^2\delta_n \sinh\,[(-1+i)d/2\delta n]} \tag{6}$$

At the surface, $x = \pm\, d/2$,

$$H_s = \frac{A}{2} - \frac{Ad}{\pi^2}\sum_{n\ \mathrm{odd}} \frac{1+i}{n^2\delta_n}\ \frac{e^{in\omega t}}{\tanh\,[(1+i)d/2\delta n]} + \frac{(-1+i)}{n^2\ \delta n}\ \frac{e^{-in\omega t}}{\tanh\,[(-1+i)d/2\delta n]} \tag{7}$$

Plots of this equation for two values of d are given in Fig. 1. The figures show that the surface field leads the average field (the dashed line) by an amount which is constant after an initial transient after field reversal. For these calculations, 100 terms are used in (7), i.e., the maximum valve of n is 199. The other parameters are A = 4, d = 109 mil or 50 mil, $\mu = 6.3 \times 10^{-3}$ T/A-m and $\rho = 5\text{x}10^{-7}$ μ m. Some other cases were run; the lead time δt for them are given in Table 1. The parameters are appropriate to M1-9 (3.75% Si) steel.

Table 1

d, mil	109	50	43.75	37.5	25
gage		18	19	20	24
δt, msec	8.68	1.69	1.30	0.95	0.42

If H_s for the 20 gage case is fitted to an equation of the form

$$H_s = a(t - T_0) + b\ e^{-t/\tau} + c \tag{8}$$

it is found that τ is about 0.2 msec. On the basis of these results, 20 gage is certainly thin enough and 18 gage might be acceptable.

The loss (watts/cubic meter) has two components, hysteretic and eddy current. The eddy current loss per unit volume W_e is given in PJ^2, where $J = \partial H/\partial x$. This must be averaged first over one time period and then over the thickness of the lamination. The result is given in Eq. 9.

$$W_e = \frac{8\ \rho A^2 d}{\pi^4} \sum \frac{1}{n^4\ \delta_n^3}\ \frac{\sin\ (d/\delta n) - \sinh\ (d/\delta n)}{\cos\ (d/\delta n) - \cosh\ (d/\delta n)}\ ,\ n \text{ odd} \tag{9}$$

Data on losses in the various electrical grades of silicon steel are usually given at 60 Hz and for peak fields of 10 and 15 kG. For a sinusoidal field, Eq. 9 reduces to

$$W_e = 8H^2_{pk}\ \frac{\rho d}{\delta^3}\ \frac{\sin d/\delta - \sinh\ (d/\delta)}{\cos d/\delta - \cosh\ (d/\delta)} \tag{10}$$

For d/δ less than 1, this can be simplified to the well known expression given in Eq. (11).

$$W_e = \rho\ H_{pk}^2\ d^2/6\delta^4 \tag{11}$$

For M-19 steel, 20 gage at 60 Hz and H_{pk} = 10 kG, Eq. (6) gives 0.32 W/lb. Data from the Metals Handbook, 8th edition (1961) for three gages, when fitted to a d^2 dependence, extrapolates to 0.36 W/lb at d = d. This good agreement gives credence to this approach.

The situation with respect to hysteretic loss is less satisfactory. The Steinmetz formula for an ac field is

$$W_h = \eta B^{1.6} \tag{12}$$

where W_h is the loss per cycle due to hystersis.
The exponent 1.6 was appropriate to steels in Steinmetz' time (1910). Modern materials have exponents from 1.5 to 2.5. Data for M-22 from the metals Handbook, at 60 Hz and 10 and 15 kg, when extrapolated to zero thickness to eliminate the eddy current portion, give an exponent of 2.3. However, this may not be applicable to the triangular waveform with dc offset. Ignoring this, and assuming a field in the iron varying between 1.5 and 4 kG, one obtains 8.7 x 10^{-4} W/lb. The field in the iron is assumed equal to the field in the gap. The eddy current loss (A = 39.7 amp/m) is 3.75 W/m^3 or 2.22 x 10^{-4} W/lb for a total loss of 1.1 x 10^{-3} W/lb. The lamination area is 0.312 m^2 and weighs 16900 lb/m^3, so the total loss is 5.8 W/m. This compares to 60 W/m in the beam tube given in Tech Note No. 4.

In summary M-19 or M-22 silicon steel with a gage of 19 or 20 would seem appropriate; the choice between the two might be determined by punchability; M-22 is less brittle. There may be difficulty in obtaining either in gages heavier than 24 (25 mil), but reducing the number of laminations by 1/3 or more is a considerable saving.

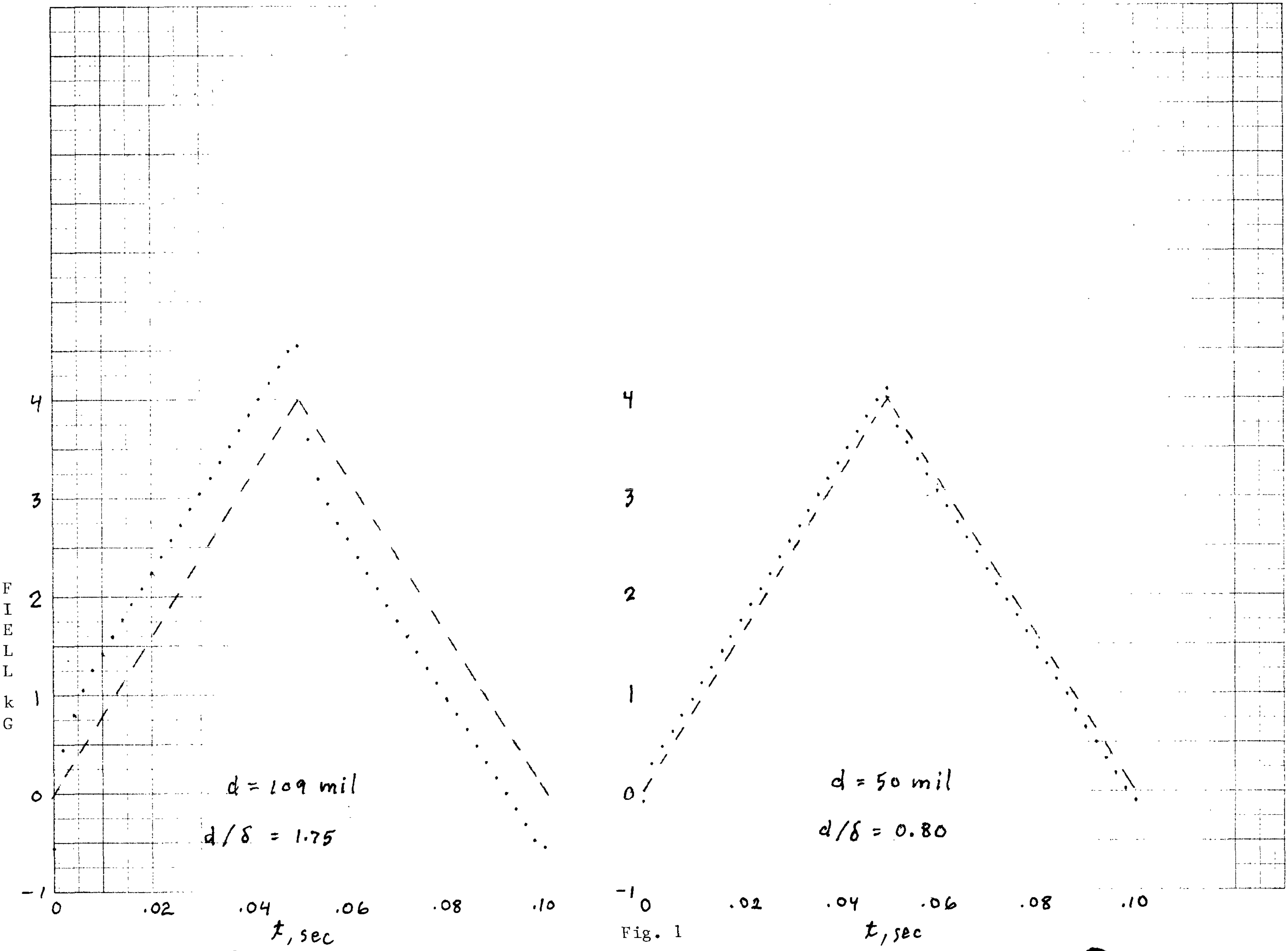
FIELD kG
4
3
2
1
0
-1
0
.02
.04
.06
.08
.10
t, sec
d = 109 mil
d/δ = 1.75
4
3
2
1
0
-1
0
.02
.04
.06
.08
.10
t, sec
d = 50 mil
d/δ = 0.80

Fig. 1